Heat Transfer to Molten Polymer Flow in Tubes

E. E. AGUR and J. VLACHOPOULOS, *Department of Chemical Engineering, McMaster University, Hamilton, Ontario, L8S 4L7 Canada*

Synopsis

This article presents the results of a numerical study (finite differences) of the heat transfer problem in flowing polymer melts. The tube wall is assumed to be at a constant temperature. The rheological behavior of the melt is described by a power law temperature-dependent model. A convective and a viscous dissipation term are included in the energy equation. Temperature profiles, bulk temperatures, and Nusselt numbers are presented for a variety of flow entry temperatures.

INTRODUCTION

In a recent article, Dang[1] presented an analytical solution to the problem of heat transfer of a power law fluid flowing inside a tube with viscous dissipation. Like most previous investigations on this topic,[2-12] Dang used a temperature-independent viscosity model. It is well known, however, that polymer melts exhibit a strong dependence of viscosity on temperature, and such a dependence is expected to influence the heat transfer characteristics. Some aspects of the heat transfer problem for a fluid with temperature and shear rate dependent viscosity have been studied by Morrette and Gogos,[13] Forsyth and Murphy,[14] Kim and Collins,[15] Forrest and Wilkinson,[16] Galili, Rigbi, and Takserman-Krozer,[17] Winter,[18,19] Popovska and Wilkinson,[20] and Pearson.[21] It should be noted, however, that none of these articles gives a complete set of results on the temperature profiles, bulk temperatures, or Nusselt numbers.

The present numerical study is for a temperature-dependent power law fluid flowing inside a tube held at a constant temperature. Results are given for a typical high-density polyethylene melt. The computer program used is available upon request.[22]

MATHEMATICAL FORMULATION AND PROCEDURE

The physical system for Poiseuille (or pressure) flow through a tube with circular cross section is illustrated in Figure 1. It consists of flow through a tube with an inside radius, a, and constant temperature walls.

The general equations for the conservation of mass, momentum, and energy[23] can be simplified by making the following assumptions:

(1) The melt is incompressible.
(2) Inertial forces are negligible in comparison with viscous and pressure forces.
(3) Body forces (e.g., gravity) are also negligible.
(4) The thermal conductivity and specific heat are constant.
(5) Normal stresses are negligible.
(6) Heat transfer by conduction in the direction of flow is negligible compared

to both convection in the direction of flow and conduction perpendicular to the direction of flow.

The simplified conservation equations for Poiseuille flow through a circular tube are

Continuity (integral form):

$$\int_{r=0}^{r=a} urdr = \frac{a^2}{2} u_{\text{avg}} \tag{1}$$

Momentum:

$$-\frac{dp}{dz} + \frac{1}{r}\frac{d}{dr}(r\tau_{rz}) = 0 \tag{2}$$

or

$$-\frac{dp}{dz} + \frac{1}{r}\tau_{rz} + \frac{d}{dr}\tau_{rz} = 0$$

Energy:

$$\rho C_p u \frac{\partial T}{\partial z} = \frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \tau_{rz}\frac{du}{dr} \tag{3}$$

or

$$\rho C_p u \frac{\partial T}{\partial z} = k\frac{\partial^2 T}{\partial r^2} + \frac{k}{r}\frac{\partial T}{\partial r} + \tau_{rz}\frac{du}{dr}$$

A common constitutive equation for polymer melts is

$$\tau_{rz} = \eta\frac{du}{dr} \tag{4}$$

where

$$\eta = Ae^{-Bn(T-T_m)}\left|\frac{du}{dr}\right|^{n-1}$$

Then, by substitution into the momentum and energy equations, we obtain

Momentum:

$$-\frac{dp}{dz} + \eta\frac{d^2 u}{dr^2} + \left(\frac{\eta}{r} + \frac{d\eta}{dr}\right)\frac{du}{dr} = 0 \tag{5}$$

Energy:

$$\rho C_p u \frac{\partial T}{\partial r} = k\frac{\partial^2 T}{\partial r^2} + \frac{k}{r}\frac{\partial T}{\partial r} + \eta\left(\frac{du}{dr}\right)^2 \tag{6}$$
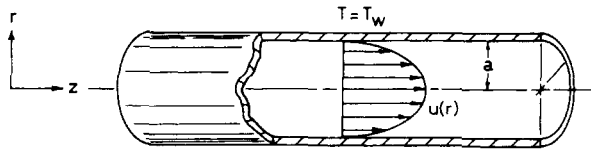

Fig. 1. Notation for Poiseuille flow through a tube of circular cross section.

where

$$\eta = Ae^{-Bn(T-T_m)} \left|\frac{du}{dr}\right|^{n-1}$$

The boundary conditions for the above equations are the following:

$$z = 0: \quad u = u_0(r) = u_{\text{avg}} \left[\frac{\nu + 2}{\nu}\right] \left[1 - \left(\frac{r}{a}\right)^{\nu}\right], \quad T = T_0$$

where

$$\nu = (n + 1)/n, \quad n = \text{power-law index} \tag{7}$$

$$r = 0: \quad du/dr = 0, \quad \partial T/\partial r = 0 \text{ (symmetry)}$$

$$r = a: \quad u = 0, \quad T = T_w$$

A $(n + 1)/n$ degree parabolic velocity profile, $u_0(r)$, and a constant temperature, $T = T_0$, have been chosen at $z = 0$. However, with the present numerical technique, any other type of starting profiles could have been used.
Let

$$u = \frac{u}{u_{\text{avg}}}$$

$$P = \frac{p - p_0}{\rho u^2_{\text{avg}}}$$

$$\theta = \frac{T - T_w}{T_0 - T_w}$$

$$Z = \frac{kz}{\rho C_p u_{\text{avg}} a^2}$$

$$R = \frac{r}{a}$$

Substituting this into eqs. (1), (5), and (6), we obtain in terms of dimensionless parameters:

Continuity (integral form):

$$\int_{R=0}^{R=1} URdR = \frac{1}{2} \tag{8}$$

Momentum:

$$-\frac{k}{C_p}\frac{dP}{dZ} + \eta\frac{d^2U}{dR^2} + \left(\frac{\eta}{R} + \frac{d\eta}{dR}\right)\frac{dU}{dR} = 0 \tag{9}$$

Energy:

$$U\frac{\partial\theta}{\partial Z} = \frac{\partial^2\theta}{\partial R^2} + \frac{1}{R}\frac{\partial\theta}{\partial R} + \gamma\left(\frac{dU}{dR}\right)^2 \tag{10}$$

where

$$\gamma = \frac{\eta u^2_{\text{avg}}}{k(T_0 - T_w)}$$

$$\eta = Ae^{-Bn(T-T_m)} \left| \frac{dU}{dR} \cdot \frac{u_{avg}}{a} \right|^{n-1}$$

The accompanying dimensionless boundary conditions are the following:

$$Z = 0: \quad U = U_0(R) = \frac{\nu + 2}{\nu}[1 - R^\nu], \quad \theta = 1$$

$$R = 0: \quad dU/dR = 0, \quad \partial\theta/\partial R = 0 \qquad (11)$$

$$R = 1: \quad U = 0, \quad \theta = 0$$

An implicit finite difference scheme of the Crank–Nicolson type was used to solve the previous equations. The procedure is explained in detail in a thesis[22] which also includes a well-documented computer program.

The local bulk temperature, T_{bulk}, and the local Nusselt number, Nu_z, were determined from the velocity and temperature profiles by using the following definitions:

$$T_{bulk} = \frac{\int_{r=0}^{r=a} T(r,z)u(r,z)rdr}{\int_{r=0}^{r=a} u(r,z)rdr} \qquad (12)$$

$$Nu_z = 2\frac{ha}{k} = \frac{-2\left(\frac{dT}{dr}\right)_{wall} a}{(T_{bulk} - T_{wall})} \qquad (13)$$

RESULTS AND DISCUSSION

Solutions of the continuity, momentum, and energy equations for Poiseuille flow through a tube with circular cross section are presented in Figures 2–7. The following velocity and temperature boundary conditions have been used:

$$z = 0: u = u_{avg}\left[\frac{\nu+2}{\nu}\right]\left[1 - \left(\frac{r}{a}\right)^\nu\right], \quad T_o = 130°C$$
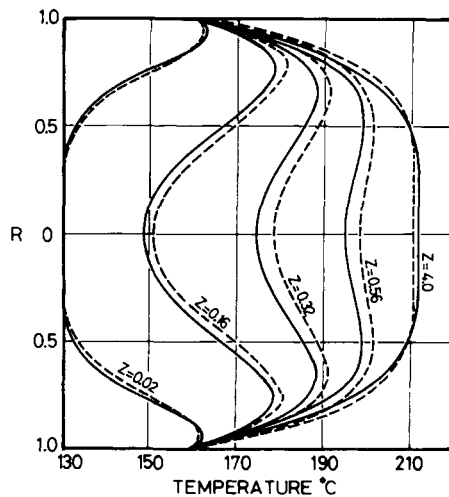
Fig. 2 Development of temperature profiles. (—) Power law temperature-dependent viscosity fluid; (- - -) temperature-independent viscosity fluid. $T_o = 130°C$; $T_w = 160°C$.
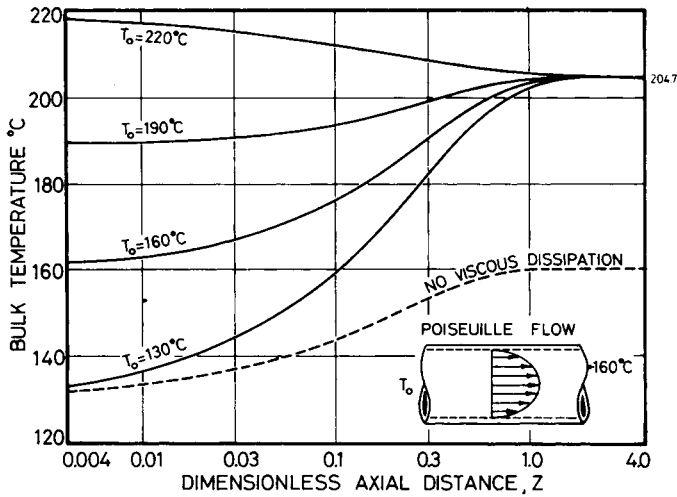
Fig. 3. Bulk temperature as a function of dimensionless axial distance. Power law temperature-dependent viscosity fluid.

where

$$u_{\mathrm{avg}} = 15.0 \text{ cm/sec}, \quad \nu = (n + 1)/n, \quad n = 0.453 \qquad (14)$$
$$r = 0: du/dr = 0, \quad \partial T/\partial r = 0$$
$$r = a = 0.125 \text{ cm}: u = 0, \quad T_w = 160°C$$

Also, the following power law temperature-dependent viscosity model and fluid properties representing a typical high-density polyethylene melt were used in the computations:

Viscosity:

$$\eta = Ae^{-Bn(T-T_m)}\left|\frac{du}{dr}\right|^{n-1}$$

where $A = 282,000$ poise·sec^{n-1} = 28,200 Pa·s^n, $B = 0.0240$ K^{-1}, $n = 0.453$, and $T_m = 399.5$ K.
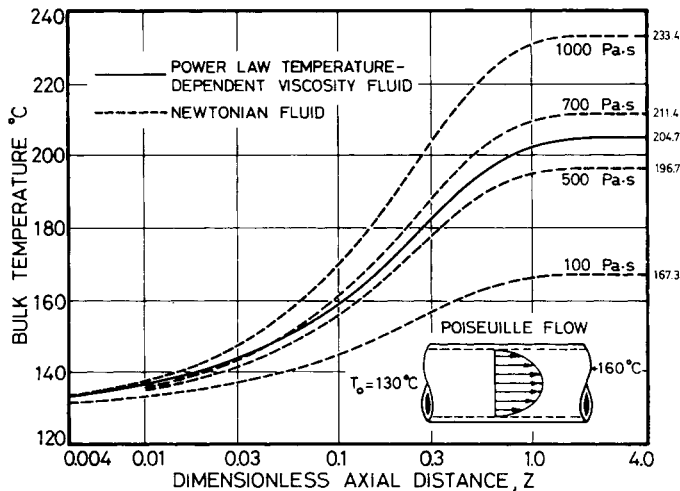

Fig. 4. Bulk temperature as a function of dimensionless axial distance. (—) Power law temperature-dependent viscosity fluid; (- - -) Newtonian fluid.
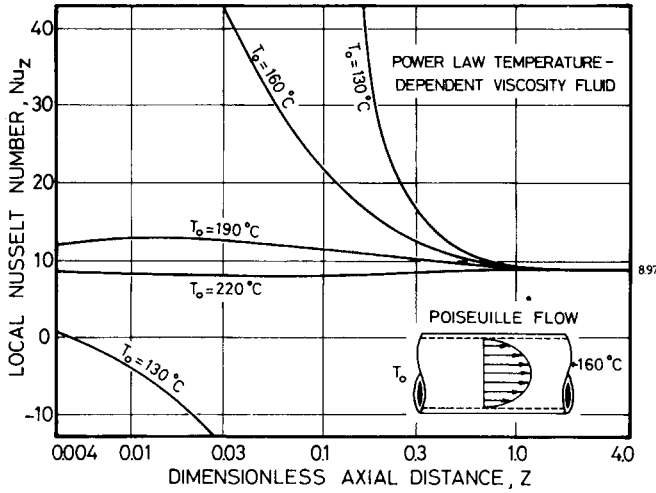
Fig. 5. Local Nusselt number as a function of dimensionless axial position. Power law temperature-dependent viscosity fluid.

Density:

$$\rho = 794 \text{ kg/m}^3$$

Specific heat: (15)

$$C_p = 0.600 \text{ cal/(g·K)}$$

$$= 2.51 \text{ kJ/(kg·K)}$$

Thermal conductivity:

$$k = 6.10 \times 10^{-4} \text{ cal/(cm·sec·K)}$$
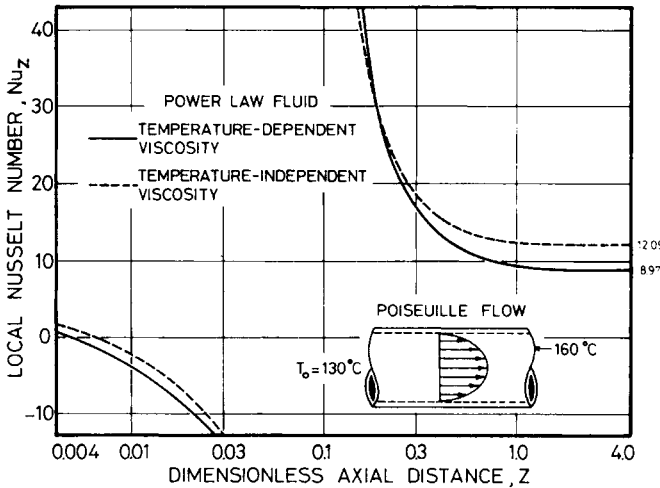
$$= 0.255 \text{ W/(m·K)}$$

Fig. 6. Local Nusselt number as a function of dimensionless axial position. (—) Power law temperature-dependent viscosity fluid; (- - -) temperature-independent viscosity fluid.
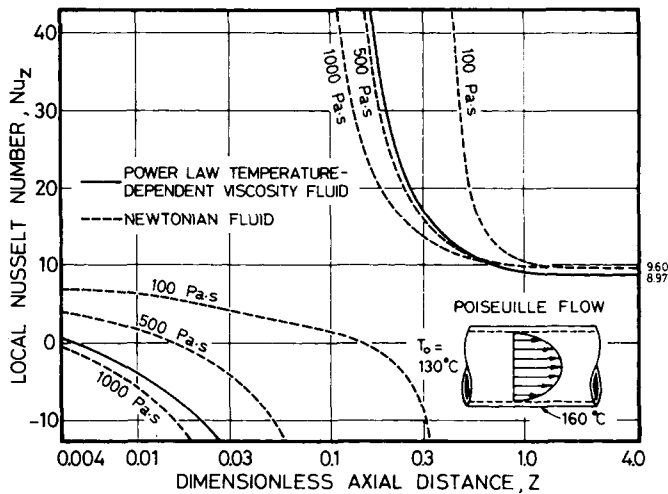
Fig. 7. Local Nusselt number as a function of dimensionless axial position. (—) Power law temperature-dependent viscosity fluid; (- - -) Newtonian fluid.

The temperature profiles, bulk temperatures, and local Nusselt numbers in Figures 2–7 are shown as functions of the dimensionless axial distance, Z.

In Figure 2, the temperature profiles for the power law temperature-dependent viscosity model and for a power law temperature-independent viscosity model are compared. The temperature-independent viscosity model used is identical to the temperature-dependent viscosity model given in eq. (15), except that T is held constant and equal to the tube-wall temperature (160°C). It can be seen that the temperature of the fluid obtained with the temperature-independent model is generally higher than is the case with the temperature-dependent model. However, the fully developed temperature profiles for the two models are about the same. At intermediate values of Z, the temperature profiles bulge near the wall, indicating that more heat is generated by viscous dissipation here than is generated near the center line of the tube. This is due to the fact that the shear rates are the highest near the tube walls.

Plots of the bulk temperatures along the length of the tube are presented in Figures 3 and 4 for the power law temperature-dependent viscosity model and for the Newtonian model. In Figure 3, the bulk temperatures are shown for power law temperature-dependent viscosity fluids with different inlet temperatures. In each case, the limiting bulk temperature is the same (204.7°C). This is to be expected since the fully developed velocity and temperature profiles are only influenced by the wall boundary conditions and by the viscosity and thermal conductivity of the fluid, but not by the inlet conditions of the fluid. Also shown in Figure 3 is the effect of removing the viscous dissipation term from the energy equation. Without viscous dissipation, the limiting bulk temperature is equal to the wall temperature (160°C). The difference of 44.7°C is an indication of the importance of viscous dissipation in the Poiseuille flow of polymer melts through a tube. The rise in bulk temperature for the power law temperature-dependent viscosity model is compared with several Newtonian models in Figure 4.

Plots of the local Nusselt number along the length of the tube are presented in Figures 5 and 6 for the power law temperature-dependent model and for the Newtonian model. In Figure 5, the local Nusselt numbers are shown for power law temperature-dependent viscosity fluids having different inlet temperatures. In each case, the limiting local Nusselt number is 8.97. Although not shown the limiting local Nusselt number for the case where viscous dissipation has been neglected is 4.00. It can be seen that when the fluid is heated by the tube walls ($T_o = 130°C$, $T_w = 160°C$), there is a region along the tube where the local Nusselt number is negative, and a point where it is discontinuous. This behavior can be easily explained in terms of eq. (13) using the value of the temperature gradient $(dT/dr)_{wall}$ and the sign of the temperature difference $T_{bulk} - T_{wall}$. A similar behavior has been also observed in a related problem.[24]

The local Nusselt numbers for the power law temperature-dependent and temperature-independent viscosity fluids are compared in Figure 6. The limiting local Nusselt numbers are 8.97 and 12.09, respectively, for the two fluids. In Figure 7, the local Nusselt numbers are shown for the power law temperature-dependent viscosity fluid and several Newtonian, constant viscosity fluids.

The results for the power law temperature-dependent viscosity model have been compared with the power law temperature-independent viscosity model and the Newtonian, constant viscosity model results. Given an appropriate temperature for the temperature-independent model, or an appropriate viscosity for the Newtonian model, it can be seen that the temperature-dependent model results are adequately estimated by the use of either of the simpler models. The choice of temperature and viscosity was made by inspection. However, if we did not have any temperature-dependent model results to compare our more simplified model results with, then we would not have anything on which to base our choice of temperature or viscosity. Furthermore, the given temperature or viscosity usually works for one type of flow only. For example, in Figure 4 it can be seen that the rise in bulk temperature for the temperature-dependent model is closely approximated by a Newtonian fluid with a viscosity of about 600 Pa·s, while in Poiseuille flow between parallel plates, a viscosity of about 700 Pa·s is required.[24]

References

1. V. D. Dang, *J. Appl. Polym. Sci.*, **23**, 3077 (1979).
2. R. B. Bird, *Chem.-Ing. Tech.*, **31**, 569 (1959).
3. R. L. Pigford, *Chem. Eng. Prog. Symp. Ser.*, **51**, (17), 79 (1955).
4. B. C. Lyche and R. B. Bird, *Chem. Eng. Sci.*, **6**, 35 (1956).
5. H. L. Toor, *Trans. Soc. Rheol.*, **1**, 177 (1957).
6. H. L. Toor, *AIChE J.*, **4**, 319 (1958).
7. J. Schenk and J. Van Laar, *Appl. Sci. Res.*, **A7**, 449 (1958).
8. W. N. Gill, *Appl. Sci. Res.*, **A11**, 10 (1963).
9. J. Sestak and M. E. Charles, *Chem. Eng. Prog. Symp. Ser.*, **64** (82), 212 (1968).
10. E. L. Smorodinskii and G. B. Froishteter, *Teor. Osnov. Khim. Tekhnol.*, **5**, 542 (1971).
11. P. Payvar, *Appl. Sci. Res.*, **27**, 297 (1973).
12. K. M. Sundaram and G. Nath, *Proc. Ind. Acad. Sci.*, **83A**, 50 (1976).
13. R. A. Morrette and C. G. Gogos, *Polym. Eng. Sci.*, **8**, 272 (1968).
14. T. H. Forsyth and N. F. Murphy, *AIChE J.*, **15**, 758 (1969).
15. H. T. Kim and E. A. Collins, *Polym. Eng. Sci.*, **11**, 83 (1971).

16. G. Forrest and W. L. Wilkinson, *Trans. Inst. Chem. Eng.*, **51**, 331 (1973).

17. N. Galili, Z. Rigbi, and R. Takserman-Krozer, *Rheol. Acta*, **14**, 816 (1975).

18. H. H. Winter, *Polym. Eng. Sci.*, **15**, 84 (1975).

19. H. H. Winter, *Adv. Heat Transfer*, **13**, 205 (1977).

20. F. Popovska and W. L. Wilkinson, *Chem. Eng. Sci.*, **32**, 1155 (1977).

21. J. R. A. Pearson, *J. Fluid Mech.*, **83**, 191 (1977).

22. E. E. Agur, "Heat Transfer in Polymer Melt Flows" M. Eng. thesis, McMaster University, Hamilton, Ont., Canada, 1978.

23. R. B. Bird, W. E. Stewart, and E. N. Lightfoot, *Transport Phenomena*, Wiley, New York, 1960.

24. E. E. Agur and J. Vlachopoulos, *Plast. Rubber Proc.*, **4**(1), 26 (March 1979).